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Finite element analysis of rectangular patch microstrip resonator with bend as a deformation

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Abstract Microstrip resonator with rectangular conducting patch has been analysed using finite Element Method (FEM) Further, the rectangular patch is bent to different angles for deformation and the variation of resonance frequencies with bending angle is tabulated It is found that bending deformation of patch, splits the degenerate states which increases with deformation

Keywords Finite element method microstrip resonator, deformation

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The resonant frequencies of open microstrip ring resonator are determined by Wolff and Menzel [1] and Pintzos and Pregla [2] The microstrip resonator with equilateral triangular patch is studied by Wolff and Knoppik [3]. Helszajn *et al* [4] and Lyon and Helszajn [5] The triangular and rectangular patch microstrip resonators have been analysed by Kalamse and Patil [6,7] Recently, Chaudhari and Patil [8] and Karkare *et al* [9] have analysed deformed microstrip resonator and bent microstrip resonator respectively, using Finite Element Method

If the resonator has got degenerate resonating frequency with different modes, then it becomes difficult to excite resonator in one particular mode To achieve it, a special provision has to be made to suppress the unwanted mode The being of the microstrip resonator may lift the degeneracy and separate resonance frequency may be obtained for each mode, so that only one wanted mode can be excited With this idea, the bent microstrip has been analysed and corresponding frequency departure is obtained The exact solution method can not apply to such structures and hence Finite Element Method has been used

A microwave resonator with conducting rectangular patch on a dielectric substrate backed by a grounded conducting plane is considered (Figure 1) In the region between the patch and ground plane, the electric field will satisfy Maxwell's equations.

$$\text{Curl Curl } \boldsymbol{E} - K^2 \boldsymbol{E} = 0.$$

Since the medium is charge-free, div $\boldsymbol{E} = 0$

$$\nabla^2 \boldsymbol{E} + K^2 \boldsymbol{E} = 0$$

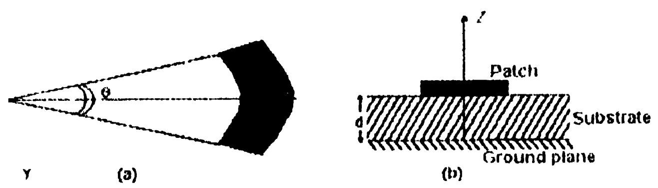

Figure 1. Rectangular patch with bend deformation (a) Top view, (b) Side view

The electric field within the substrate has only z-component and magnetic field has x- and y-components Further, if the thickness of the substrate d is much small as compared to wavelength of electromagnetic waves, then the fields will not vary along Z direction

The tangential component of magnetic field at the edge is negligible and we can consider the vertical sides of the substrate at the edges of the patch as magnetic walls. Hence, the fields within the resonator corresponding to TM modes will be generated by the equation

$$\nabla^2 \boldsymbol{E} + K^2 \boldsymbol{E} = 0 \quad (1)$$

with the boundary conditions

$$\frac{\partial E_z}{\partial n} | B1 = 0 \quad (2)$$

$$E_Z | B2 = 0 \quad (3)$$

where E_Z is z-component of E, $\partial/\partial n$ represents normal derivatives, B_1 consists of side boundary surfaces, B_2 consists of top patch and part of ground plane below the patch. The domain Ω of resonator is bounded by B_1 and B_2.

To get the expression for the functional Π, multiply eq. (1) by some weighting function V^* and integrate it over the domain of the resonator.

$$\Pi = \iiint_{\Omega} V^* \operatorname{div}(\operatorname{grad} Ez)\, d\Omega + K^2 \iiint_{\Omega} V^*\, Ez\, d\Omega. \tag{4}$$

The first term is of the type S div$A$ and we can replace it by div(SA) − grad$(S).A$.

$$\Pi = \iiint_{\Omega} -(\operatorname{grad} V^*).\operatorname{grad} Ez\, d\Omega + \iiint_{\Omega} \operatorname{div}(V^* \operatorname{grad} Ez)\, d\Omega + K^2 \iiint_{\Omega} V^*\, Ez\, d\Omega. \tag{5}$$

Application of Gauss divergence theorem to the second term in the above equation leads to

$$\Pi = \iiint_{\Omega} (\operatorname{grad} V^*).\operatorname{grad} Ez\, d\Omega + \iiint_{\Omega} (V^* \operatorname{grad} Ez.n.ds) + K^2 \iiint_{\Omega} V^*\, Ez\, d\Omega. \tag{6}$$

The last integral can be written separately for B_1 and B_2. But it is clear that due to the condition of eq. (2), the integral over B_2 will vanish. The condition (2) is a natural condition while (3) is an essential condition. Further, it is worth noting that the variation of fields along Z direction is absent and hence the integration over Z will not contribute a constant term which is not important from variational point of view. Hence, we transform the volume integral over the surface which is bounded by the boundary of the patch.

$$\Pi = \iint |\operatorname{grad} Ez|^2\, dx\, dy + K^2 \iint |Ez|^2\, dx\, dy, \tag{7}$$

where V^* is replaced by Ez and negative sign of Π is neglected.

For the boundaries B_1, the normal is parallel to Z-axis and (grad Ez. n) is $\partial Ez / \partial z$. Since there is no variation of fields along Z-axis, the contribution from the term at B_1 is zero.

The rectangular area of the patch is divided into triangular as well as quadrilateral elements. The triangular elements are used at the edges only. The mapping functions assumed for these elements are quadratic in nature. The triangular elements and quadrilateral elements are taken with six and eight nodes, respectively.

$$\Pi = \sum \{Ez^e\}^T [S^e] \{Ez^e\} + K^2 \sum_{lm} \{Ez^e\}^t [T^e] \{Ez^e\} \tag{8}$$

Where $$S_{ij}^e = \left[\frac{\delta F_i}{\delta x}\frac{\delta F_j}{\delta x} + \frac{\delta F_i}{\delta y}\frac{\delta F_j}{\delta y} + \frac{\delta F_i}{\delta z}\frac{\delta F_j}{\delta z} \right] dx\, dy \tag{9}$$

and $$T_{ij}^e = \int F_i F_j\, dx\, dy. \tag{10}$$

Here, F_i is the mapping function due to i-th node of an element surface.

The condition that variation of Π must be zero gives the matrix equation in terms of assembled matrices S and T as

$$[S]\{Ez\} + K^2 [T]\{Ez\} = 0, \tag{11}$$

An electromagnetic resonator with rectangular patch has been analysed to know the variation of resonating frequencies with the efect of bending of the patch in the plane of the patch. The size of the patch considered is 1 cm × 1 cm. In this case, the area of the patch is kept constant during the process of bending. The different bendings are considered by varying the radius of curvature. The calculated resonant frequencies are given in Table 1.

Table 1. Resonant K^2 values for different angles subtended by the bending of a rectangular patch

No Deformation	Angle through which patch is bent 10°	20°	30°	40°	50°	60°
14 99	14 89	14 65	14 33	13 97	13 59	13 25
14 99	15 05	15 21	15 48	15 85	16 32	16 88
21 21	21 39	21 92	22 73	23 73	24 81	25 15
30 04	29 46	28.40	27 35	26 42	25 65	25 92
30 04	30 13	30 41	30 89	31 57	32 45	33 51
33 58	33 67	33 97	34 43	34 94	35 19	35 11

It is observed that the bending deformation of the patch splits the degenerate states, one frequency of which decreases with increase of bending while the other increases. The separation between the split frequencies increases with deformation. This separation differs from one degenerate frequency to another degenerate frequency. For any bending, the separation is higher for higher frequencies. The nondegenerate frequencies show increase in their value with deformation.

Acknowledgment

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